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LETTER TO THE EDITOR

Lattice-induced anisotropy in a diffusion-limited growth model

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Abstract. We have investigated the diffusion-limited aggregation process on a square lattice by solving the Laplace equation numerically. In contrast to previous models, at each time step the occupancy probability is used directly for the growth of all of the perimeter sites. We find that regardless of the initial shape of the seed particles, the final pattern of the aggregate has an anisotropic shape. We have interpreted the pronouncement of the lattice-induced anisotropy as arising from averaging of the shape fluctuations in this growth process. The values of the fractal dimension of the clusters are found to agree with the predictions of Turkevich and Scher implying that the clusters in our model have the same structure as large DLA and dielectric breakdown clusters.

The study of aggregation and growth processes has become a subject of increasing interest in recent years [1–4]. Much of this interest has been stimulated by the diffusion-limited aggregation (DLA) model introduced by Witten and Sander [5]. The simple, yet fundamental, nature of the diffusion-limited aggregation process coupled with its complex and mathematically intractable features have made DLA one of the central models for the investigations of non-equilibrium growth and aggregation processes [1–4]. DLA clusters are fractals [6] and their fractal dimension D can be regarded as a critical exponent describing a non-trivial power-law divergence with the linear size of the aggregate. It is for this reason that the very recent suggestions [7, 8] that the DLA fixed point is unstable with respect to the symmetry of the underlying lattice on which it is investigated has generated much interest. This is the strongest evidence to data for the dependence of *any* critical exponent on the lattice structure and therefore deserves detailed investigation. In addition to the possible non-universality of D it was also found that the shape of large DLA [8–11] and Eden [12] clusters is anisotropic, i.e. it depends on the underlying lattice. This kind of anisotropy is rooted in the growth mechanism itself and is different from the presumably lattice-independent anisotropy of large lattice animals and percolation clusters found by Family *et al* [13].

In this letter we study a model closely related to diffusion-limited aggregation processes on a square lattice by calculating the cluster growth probability on the interface by solving the Laplace equation numerically. In contrast to previous studies [14] based on the solution of the Laplace equation, in our model at each time step

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the growth takes place with a certain probability at all perimeter sites. We find that the addition of several particles at each growth step leads to an averaging of the shape fluctuations and the aggregates grown in this manner are more regular than those generated with the random addition of a single particle at each stage. This is clearly seen by the fact that the clusters in our model take on the symmetry of the underlying structure of the lattice much faster than in the other models, regardless of the initial shape of the seed cluster. Moreover, despite the change in the cluster shape, we find the fractal dimension of the clusters to agree with those predicted by Turkevich and Scher [7] for DLA [5] and the dielectric-breakdown-type [14] clusters on a square lattice. This indicates that the clusters in our model have the same structure as very large clusters in these models grown on a square lattice.

It was pointed out by Witten and Sander [5] that the random walk problem in DLA satisfies the equation $\Delta u(\boldsymbol{r}, t)=0$, where $u(\boldsymbol{r}, t)$ is the concentration of the random walkers at the vicinity of point $\boldsymbol{r}$ at time t. Since the perimeter sites are perfect traps and the walkers cannot penetrate the cluster, $u=0$ on the cluster. There is also an additional boundary condition that $u(r\to\infty)$, because random walkers are isotropically launched from infinity. The probability $p(\boldsymbol{r})$ that a walker is absorbed at the perimeter site $\boldsymbol{r}$ is proportional to the flux $\boldsymbol{\Delta} u$. Thus, the DLA problem maps onto an electrostatic problem [14, 15] where u is the potential obtained by solving the Laplace equation with the boundary conditions $u=0$ on the conductor and $u=1$ on a circle of infinite radius. Indeed, the patterns found in the experiments on dielectric breakdown [14] and electrodeposition [15, 16] are very similar to the computer-generated DLA clusters.

One obtains a generalisation of the above model if one assumes that $p(\boldsymbol{r})$ is given by [7, 14]

$$p(\boldsymbol{r})=\frac{|\boldsymbol{E}(\boldsymbol{r})|^{\eta}}{\int_{\pi}|\boldsymbol{E}(\boldsymbol{r})|^{\eta}\,\mathrm{d}s} \tag{1}$$

where the electric field $\boldsymbol{E}=-\boldsymbol{\nabla} u$ and η is an adjustable parameter. In (1), the probability density is normalised over the cluster perimeter π. Turkevich and Scher [7] have recently calculated the occupancy probability $p(\boldsymbol{r})$ for diffusion-limited aggregation on a square lattice assuming that large clusters have a diamond-shaped envelope. They further argued that the growth rate of the large DLA is independent of the details of the cluster interface and from the singularity of the occupation probability they found that the fractal dimension D of the clusters on a square lattice is given by [7]

$$D=2-(\eta/3). \tag{2}$$

One of the objectives of the present letter is to test the above prediction for our growth model.

We have solved the Laplace equation on a square lattice with a circle of radius $R_{\max}$ maintained at constant unit potential and a single seed particle or a cluster of particles placed at the origin at a potential of zero. In each growth step, the perimeter sites are occupied with the probability $(|\boldsymbol{E}|/E_{\max})^{\eta}$ where $\boldsymbol{E}$ is the gradient of the potential at the growth site and $E_{\max}$ is the maximum value of $|\boldsymbol{E}|$ at the given growth stage. Thus, sites having the maximum value of the gradient are always occupied, but other perimeter sites grow with a smaller probability. This model corresponds to those experimental situations where the growth occurs along the surface simultaneously with varying probability. After all interface sites have been examined for possible occupancy, the potential u is 'relaxed' by replacing u on each lattice site by the average of the potential on the four nearest-neighbour lattice sites. The iterations are stopped when they change the value of u at a site by less than one part in a thousand. After the iterations are stopped, i.e. the potential u is relaxed, all surface sites are again examined for possible occupancy. First a random number in the range $0<x<1$ is generated. If $x<(|E|/E_{max})^{\eta}$, then the site is occupied. The sequence of relaxation and growth is repeated until a large cluster has grown.

The typical patterns obtained from our model for $\eta = 1.00$ and 2.00 are shown in figures 1(a) and 1(b) respectively. Figure 1 should be compared with ordinary DLA clusters [5] on a square lattice and both figures should be compared with the dielectric breakdown patterns [14] for the corresponding values of η on a square lattice. Clearly the patterns in our model are much more regular and more anisotropic than clusters of the same size in the other models. The regularity of the patterns and the pronounced anisotropy of their shapes arises from the smoothing or the averaging effect of adding several particles to the cluster at each growth step with probabilities that are calculated for each site. Similar results have recently [11] been found for DLA clusters. Clearly as η increases more weight is given to the 'hottest' tips and the cluster becomes more stringy. Figure 1(b) is similar to figure 16 of Chen and Wilkinson [17] who solved the Laplace equation for the case of viscous flow in a network of channels and introduced the randomness into the medium by varying the conductivity of the channels.

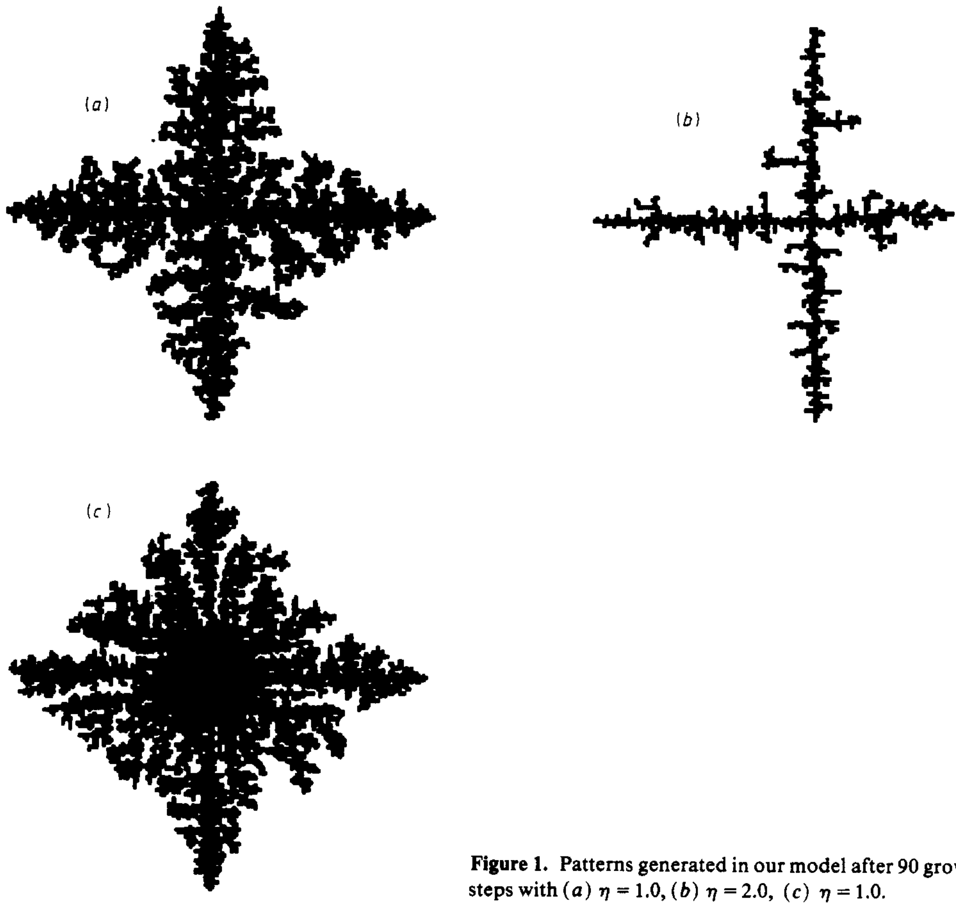

Figure 1. Patterns generated in our model after 90 growth steps with (a) $\eta = 1.0$, (b) $\eta = 2.0$, (c) $\eta = 1.0$.

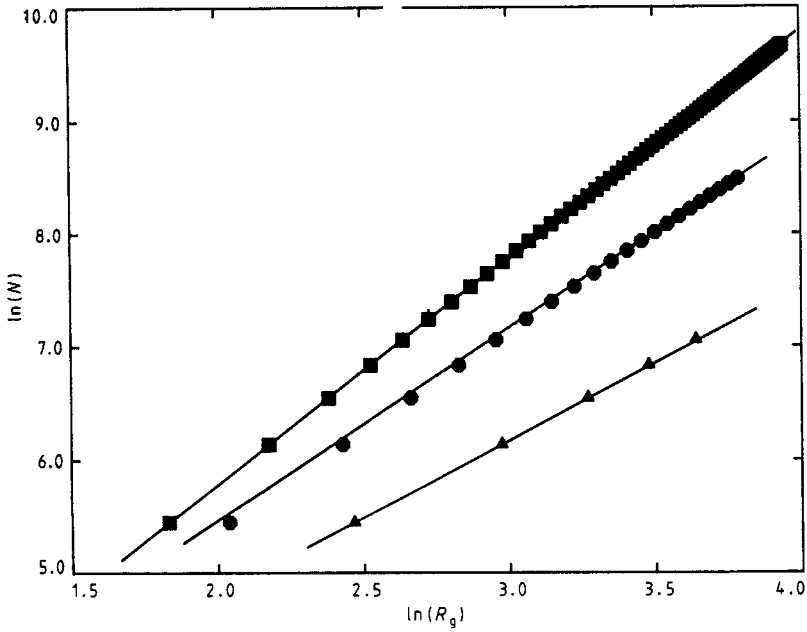

Figure 2. Log-log plot of the cluster mass N against the radius of gyration R_g, averaged over ten simulations, each having 90 growth steps, for $\eta = 0.05$ (■), 1 (○) and 2 (▲). The slopes of the straight lines through the data points are 1.95, $\frac{5}{3}$ and $\frac{4}{3}$, calculated from relation (1) for $\eta = 0.05$, and 1 and 2, respectively.

To show that the anisotropy of the clusters is a general feature of the growth on a square lattice, in figure 1(c) we have studied the growth with a number of particles in a circularly shaped seed. From the simulations we see that the underlying shape and symmetry of the seed is rapidly lost by a few growth steps leading to anisotropic patterns like figures 1(a) and 1(b).

In order to study the scaling properties of the clusters and to determine their fractal dimension we have calculated the dependence of the cluster mass N on the radius of gyration R_g. The results averaged over ten simulations, each having 90 growth steps, are shown in figure 2 for $\eta = 0.05$, 1 and 2. The slopes of the straight lines drawn through the data points are 1.95, $\frac{5}{3}$ and $\frac{4}{3}$ and are based on the prediction of Turkevich and Scher [7] for D given in relation (1) for $\eta = 0.05$, 1 and 2, respectively, and fit the data quite well. We have also calculated D by determining the dependence of the average number of filled sites $\bar{N}$ in a box of length L centred about the central site on the cluster. The results for $\eta = 0.05$, 1 and 2 are shown in figure 3 where log $\bar{N}$ is plotted against log L. Again, the slopes of the straight lines in figure 3 are 1.95, $\frac{5}{3}$ and $\frac{4}{3}$ and are calculated from relation (1) and fit the data quite well. The results of figures 2 and 3 indicate that although the clusters in our model have a highly anisotropic structure, they still have the values of the fractal dimension directly calculated from simulations of DLA [5] and dielectric breakdown [14] clusters on a square lattice. This suggests that the clusters grown in our model have the same structure as much larger clusters in the other models grown on a square lattice.

The anisotropic shape of the clusters is due to the structure of the underlying lattice [7-11]. This lattice anisotropy can, however, be compensated by the fluctuations due

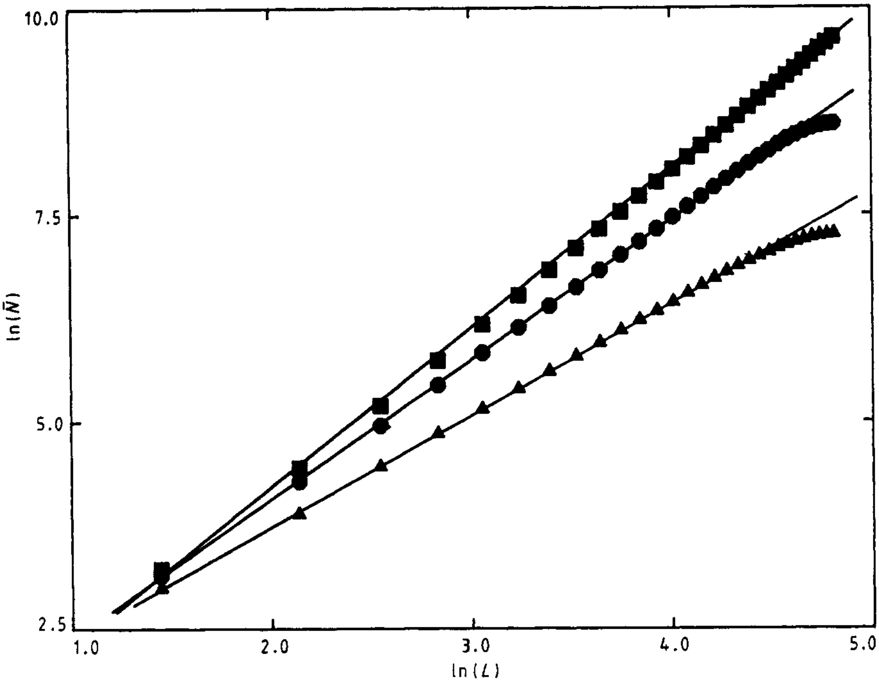

Figure 3. Log-log plot of the mean number of particles $\bar{N}$ within a box of length L, against L, averaged over ten simulations, each having 90 growth steps, for $\eta = 0.05$ (■), 1 (○) and 2 (▲). The slopes of the straight lines through the data points are 1.95, $\frac{5}{3}$ and $\frac{4}{3}$, calculated from relation (1) for $\eta = 0.05$, 1 and 2, respectively.

to the randomness of the growth process by various extents [11, 17]. In the original version of DLA [5] and the dielectric breakdown model [14] one particle is added to the growing cluster at each time step and the fluctuations dominate the growth more effectively than in our case. This is so because in our model the probabilities $p(\boldsymbol{r})$ at a given time step t_0 do not depend on the position of the newly (at t_0) added particles. This results in a decrease of fluctuations because the probability of filling a site is not perturbed by the random event whether the neighbouring sites were filled at t_0. As a result some of the short length scale fluctuations are averaged out like in a real space renormalisation method.

In conclusion, we have studied the diffusion-limited aggregation growth process on a square lattice by solving the Laplace equation numerically. In contrast to previous studies [14], in our model the occupancy probability is used directly for growth of every site on the interface. We find that regardless of the initial shape of the seed particles, the final pattern of the aggregate has an anisotropic shape. We have interpreted the anisotropy as arising from the structure of the underlying lattice due to the averaging of the shape fluctuations. The value of the fractal dimension of the clusters is found to agree with the predictions of Turkevich and Scher [7], implying that our aggregates are similar to very large DLA [5] and dielectric breakdown clusters [14].

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Note added in proof. Recently, Family *et al* [18] have analytically studied the effects of an m-fold anisotropy, induced by the growth mechanism or the lattice symmetry, on the asymptotic structure of two-dimensional DLA clusters. They find $D = 1 + (1 - \nu_\parallel)/\nu_\perp$, $\nu_\parallel = \frac{2}{3}$ and $\nu_\perp = 2(m-1)/3m$, where $\nu_\parallel$ and $\nu_\perp$ are the exponents describing the divergence of lengths parallel and perpendicular to the direction of anisotropy, respectively.

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